



Oxford Cambridge and RSA

GCE

Further Mathematics A

Y534/01: Discrete Mathematics

AS Level

Mark Scheme for June 2023

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS**PREPARATION FOR MARKING
RM ASSESSOR**

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal <http://www.rm.com/support/ca>
3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. Subject Specific Marking Instructions

- a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

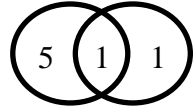
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

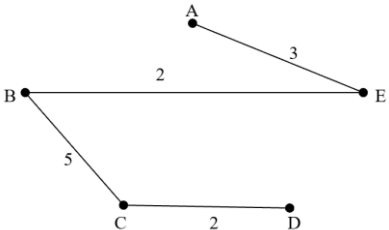
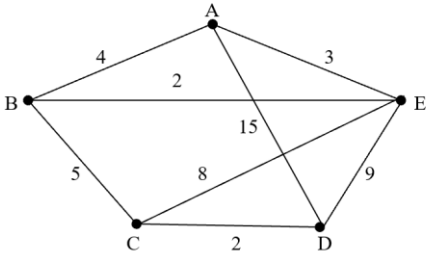
- f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
- NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads “2 s.f”.

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.



- g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

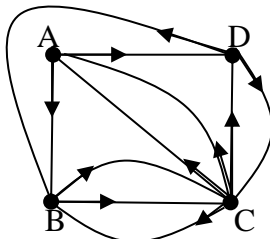
Question		Answer	Marks	AO	Guidance
1	(a)	Find a bus or train (or both) that can be used to travel from home to town	B1 [1]	1.1	Any appropriate problem, request or question, in context, that would require a construction for its solution
1	(b)	$7 = 6 + 2 - n(B \cap T)$ $8 - 7$ 1 journey by both bus and train	M1 A1 [2]	2.5 2.2a	Use inclusion-exclusion (soi) Maybe shown as a Venn diagram o.e.  Bus Train 1 seen as answer (not picked out from Venn diagram, unless clearly indicated)

Question		Answer	Marks	AO	Guidance																																									
2	(a)	<p style="text-align: center;">A <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>1</td><td>0</td></tr><tr><td> </td><td> </td></tr></table></p> <p>B <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>3</td><td>4</td></tr><tr><td>4</td><td> </td></tr></table> E <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>2</td><td>3</td></tr><tr><td> </td><td>3</td></tr></table></p> <p>C <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>9</td></tr><tr><td>11</td><td>9</td></tr></table> D <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>5</td><td>11</td></tr><tr><td>15</td><td>12</td><td>11</td></tr></table></p> <p>Shortest path: A – B – C – D</p>	1	0			3	4	4		2	3		3	4	9	11	9	5	11	15	12	11	<p>M1</p> <p>A1</p> <p>B1</p> <p>[3]</p>	<p>3.1a</p> <p>1.1</p> <p>1.1</p>	<p>Using Dijkstra, a valid updating seen</p> <p>Permanent values all correct</p> <p>Or from D:</p> <p>A <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>5</td><td>11</td></tr><tr><td>15</td><td>11</td></tr></table> B <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>3</td><td>7</td></tr><tr><td> </td><td>7</td></tr></table> C <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>2</td><td>2</td></tr><tr><td> </td><td>2</td></tr></table> D <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>1</td><td>0</td></tr><tr><td> </td><td> </td></tr></table> E <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td>4</td><td>9</td></tr><tr><td> </td><td>9</td></tr></table></p> <p>A – B – C – D (not in reverse)</p>	5	11	15	11	3	7		7	2	2		2	1	0			4	9		9
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2	(b)	<p>BE = 2 ✓ CD = 2 ✓ AE = 3 ✓ AB = 4 BC = 5 ✓ CE = 8 DE = 9 AD = 15</p> <p>Minimum spanning tree: BE, CD, AE, BC</p> 	<p>M1</p> <p>A1</p> <p>B1</p> <p>[3]</p>	<p>1.1</p> <p>1.1</p> <p>1.1</p>	<p>Using <u>Kruskal</u>, list of (at least the first 5) arcs (and weights) in increasing order of weight, o.e. Allow BE, CD, AE, BC listed in order, with no others BE and CD may be interchanged</p> <p>Not choosing arc AB in list AB indicated in a different way from BE etc Allow AB completely missing from list</p> <p>Choosing arcs BE, CD, AE, BC (only), allow A-E-B-C-D From a correct tree seen or arcs listed (separately from Kruskal working) (need not show arc weights) For reference:</p> 																																									

Question			Answer	Marks	AO	Guidance
3	(a)	(i)	First pass 10 18 7 23 54 31 62 (82)	M1	1.1	Bubble increasing Result at end of first pass starts 10 18 7 ...
			Second pass 10 7 18 23 31 54 (62 82)	A1	1.1	Results correct at end of first, second and third passes (cao)
			Third pass 7 10 18 23 31 (54 62 82)	A1	1.1	A <u>final</u> fourth pass in which nothing changes
			Fourth pass 7 10 18 23 31 54 62 82			Allow at most one misread for max M1 A0 A1 Decreasing: first pass starts 23 18 10 ... ⇒ M1 A0 A0
				[3]		
3	(a)	(ii)	First = 5	M1	1.1	5 swaps in first pass (allow FT provided M1 gained in (a)(i))
			Second = 2, third = 1	A1	1.1	2 swaps in second pass and 1 swap in third pass (cao)
			Fourth pass = 0			Ignore swaps for fourth pass (and any others that are shown) Swaps must be written in figures (not tallies)
				[2]		
3	(b)	(i)	First pass 10 23 18 7 62 54 31 82	M1	1.1	Shuttle increasing Result at end of first pass starts 10 23
			Second pass 10 18 23 (7 62 54 31 82)	A1	1.1	Results correct at end of second, third and fourth passes (cao)
			Third pass 7 10 18 23 (62 54 31 82)	A1	1.1	Results correct at end of fifth, sixth and seventh passes (cao)
			Fourth pass 7 10 18 23 62 (54 31 82)			Seven passes used in total
			Fifth pass 7 10 18 23 54 62 (31 82)			Allow at most one misread for max M1 A1 A0
			Sixth pass 7 10 18 23 31 54 62 (82)			Decreasing: first pass starts 23 10 with 0 swaps ⇒ M1 A0 A0
			Seventh pass 7 10 18 23 31 54 62 82			
				[3]		

Question			Answer	Marks	AO	Guidance
3	(b)	(ii)	First = 1, second = 1, third = 3, fourth = 0	M1	1.1	Number of swaps correct for first four passes (allow FT provided M1 gained in (b)(i) and consistently increasing, or decreasing)
			Fifth = 1, sixth = 2, seventh = 0	A1	1.1	All correct, with exactly 7 passes used Swaps must be written in figures (not tallies)
				[2]		
3	(c)		E.g. Both use 8 swaps but shuttle sort uses fewer comparisons so is more efficient	B1	3.1a	Fewer comparisons or smaller total (from valid reasoning and increasing) Number of comparisons need not be evaluated (or be correct) (bubble = 7+6+5+4 = 22, shuttle = 1+2+3+1+2+3+1=13) but shuttle sort must have fewer comparisons than bubble sort (their) 13 < (their) 22 or (their) 21 < (their) 30 Not using ratios
				[1]		
3	(d)		E.g. Takes (approximately) 9 times as long	B1	2.2a	3^2 or 9 seen in context, $\left(\frac{3000}{1000}\right)^2 = 9$ (so run time is) 9 times as long Allow 'increase by 9' (BOD scale factor), allow $\times 9$ 'exponential' \Rightarrow B0 unless $\times 9$ (o.e.) also seen
				[1]		

Question			Answer	Marks	AO	Guidance
4	(a)	(i)	In complete graph K_4 each vertex has degree 3	B1	2.4	Complete (or K_4) degree 3, allow odd Diagrams are not sufficient explanation
			In (a simply connected) Eulerian graph (with 4 vertices) each vertex has even degree so there is a contradiction	B1	2.4	Eulerian (or G) no odd (or all even), allow degree 2 o.e Allow 'some even' or (at most) two odd (i.e. semi-Eulerian) Diagrams are not sufficient explanation Allow K_4 has odd vertex degrees so not Eulerian for B1 B1
4	(a)	(ii)	Number of arcs in graph G is 4 G is Eulerian so vertex degrees are even G is connected so vertex degrees are not 0 G is simple so no vertex degree > 3 Hence each vertex of G has degree 2	B1 B1 [2]	2.2a 2.4	4 Explaining the significance of G being (Eulerian,) <u>simple</u> (and connected) e.g. stating that each degree < 4 , not just 'no loops' and hence each vertex has degree 2 Diagrams are not sufficient explanation
4	(a)	(iii)	Each vertex of G is directly connected to (only) 2 of the others Use vertices that are not directly connected to form the two sets.	B1 [1]	1.1	Showing that G is bipartite (no FT) This could be shown in a diagram provided the two sets are identified e.g. by drawing a ring round the vertices or from labels (set 1 and set 2 o.e.) If no indication of the sets, a diagram  or  with $K_{2,2}$ stated

Question			Answer	Marks	AO	Guidance																																	
4	(b)	(i)	The leading diagonal has all zero entries	B1 [1]	2.2a	No arcs for each vertex to itself Allow 'A – A = 0 etc.' o.e. (with B, C, D implied from etc.) Diagrams are not sufficient explanation																																	
4	(b)	(ii)	The sum of the entries in the matrix is 10	B1 [1]	2.2a	1+ 1+ ... = 10 or 2 + 2 + 4 + 2 = 10, or similar Allow 10 (without calculation)																																	
4	(b)	(iii)		B1 [1]	1.1	This graph o.e. with 10 directed arcs For reference: <table border="1" data-bbox="1288 662 2083 885"> <thead> <tr> <th colspan="2"></th> <th colspan="4">To</th> </tr> <tr> <th colspan="2"></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> </tr> </thead> <tbody> <tr> <th rowspan="4">From</th> <th>A</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>B</th> <td>0</td> <td>0</td> <td>2</td> <td>0</td> </tr> <tr> <th>C</th> <td>2</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>D</th> <td>0</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>			To						A	B	C	D	From	A	0	1	0	1	B	0	0	2	0	C	2	1	0	1	D	0	1	1	0
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	D	0	1	1	0																																		
4	(b)	(iv)	e.g. C – A – B – C – A – D – C – D – B – C – B	M1 A1 [2]	1.1 1.1	Trail starts at C and ends at B (not reversed) and includes every letter at least once A valid trail (using each directed arc exactly once, no FT) that passes through A, B and D (exactly) twice and passes through C three times 2 A's, 3 B's, 4 C's, 2 D's																																	

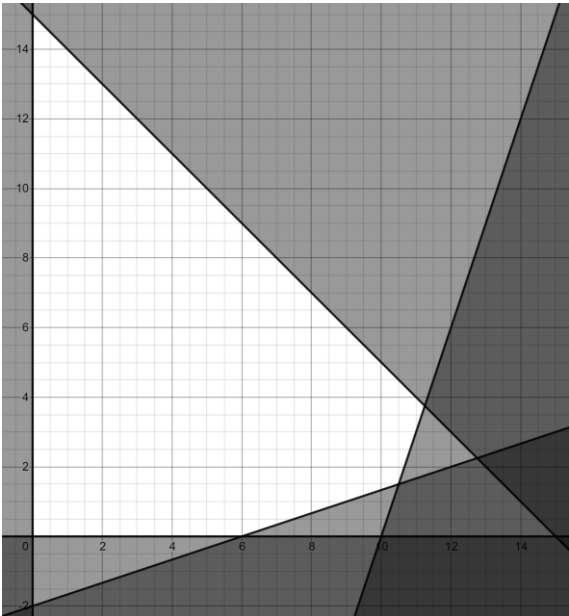
Question		Answer	Marks	AO	Guidance
5	(a)	<p>5 (hours)</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>3.4</p> <p>1.1</p>	<p>Forward pass seen (first value in bold at each vertex) Allow 0 and/or 5 missing but otherwise correct</p> <p>cao 5 stated (not implied from diagram)</p>
5	(b)	<p>See answer given in part (a)</p> <p>A, B, C, D, E (in any order)</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>3.4</p> <p>1.1</p>	<p>Backward pass seen (second value in bold at each vertex) Allow 0 and/or 5 missing but otherwise correct</p> <p>cao</p>

Question			Answer	Marks	AO	Guidance
5	(c)		$F = 3 - 0.5 - 1 = 1.5$ (hours) $G = 4.5 - 1.5 - 1.5 = 1.5$ (hours) $J = 5 - 3 - 0.5 = 1.5$ (hours) $H = 3.5 - 0.5 - 2 = 1$ (hour) $I = 5 - 2.5 - 1.5 = 1$ (hour)	M1 ft A1 A1	1.1 1.1 1.1	Any two correct, or from <u>their</u> forward and backward passes Working need not be seen F, G, J = 1.5 (cao) Working need not be seen H, I = 1 (cao) Working need not be seen
5	(d)	(i)		B1 B1	3.3 3.5c	C, E, G, I correct Activity L is after C, E, G and I (directions may be implied) Activity L is before J (directions may be implied)
5	(d)	(ii)	Time to start of L = 5 hours So L + J take 2.5 hours Duration of L = 2 hours	B1 B1	3.4 2.2a	Appropriate working seen, with evidence of what the values represent e.g $A + B + C + L + J = 7.5$ or $A + D + E + L + J = 7.5$ o.e. use of critical path No FT 2 (cao)

Question		Answer	Marks	AO	Guidance																														
6	(a)	Delete W	B1	2.2a	From column <u>headings</u> in a reduced table Ignore if any rows also deleted In each row, value in col W > value in col Y So col W is dominated by col Y or col Y dominates col W																														
		Delete Z	B1	2.2a	From column <u>headings</u> in a reduced table Ignore if any rows also deleted In each row, value in col Z > value in X So col Z is dominated by col X or col X dominates col Z																														
		<table border="1"> <thead> <tr> <th></th> <th>X</th> <th>Y</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>0</td> <td>2</td> </tr> <tr> <td>B</td> <td>2</td> <td>-3</td> </tr> <tr> <td>C</td> <td>4</td> <td>-4</td> </tr> <tr> <td>D</td> <td>-1</td> <td>5</td> </tr> </tbody> </table>		X	Y	A	0	2	B	2	-3	C	4	-4	D	-1	5			<p>Alternative method</p> <table border="1"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> </tr> </thead> <tbody> <tr> <td>X</td> <td>0</td> <td>-2</td> <td>-4</td> <td>1</td> </tr> <tr> <td>Y</td> <td>-2</td> <td>3</td> <td>4</td> <td>-5</td> </tr> </tbody> </table> <p>Or transposed</p>		A	B	C	D	X	0	-2	-4	1	Y	-2	3	4	-5
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D	6	-1	5	0																															

Question		Answer	Marks	AO	Guidance
6	(b)	X has probability p and Y has prob $1 - p$ A: $0p + 2(1 - p) = 2 - 2p$ B: $2p - 3(1 - p) = 5p - 3$ C: $4p - 4(1 - p) = 8p - 4$ D: $-p + 5(1 - p) = 5 - 6p$	M1 ft	3.1a	Definition of p may be implied for M marks Using their reduced table (or correct) and their (single) variable At least two correct expressions (need not be simplified) seen or implied from graph Using values from original table means that these are losses for Casey, so will need the minimax (lowest point of upper boundary)
		Alternative method (change signs) X has probability p and Y has prob $1 - p$ A: $0p - 2(1 - p) = -2 + 2p$ B: $-2p + 3(1 - p) = -5p + 3$ C: $-4p + 4(1 - p) = -8p + 4$ D: $p - 5(1 - p) = -5 + 6p$	M1 ft		Definition of p may be implied for M marks Using their reduced table (or correct) and their (single) variable At least two correct expressions (need not be simplified) seen or implied from graph Using negatives of values from original table means that these are gains for Casey, so will need the maximin (highest point of lower boundary)
		$8p - 4 = 5 - 6p$ $p = \frac{9}{14}$ Optimal mixed strategy: choose randomly between X and Y, choosing X with prob $\frac{9}{14}$ and Y with prob $\frac{5}{14}$	M1 ft A1 A1 [4]	3.1a 1.1 3.2a	Solving for (their) p (graphically or algebraically or implied) $p = 0.643$ (or better) (0.6428571...) (cao) Correct probabilities described in context involving <u>both X and Y</u> Or correct ratios o.e. e.g randomly choose X 1.8 times as often as Y

Question			Answer	Marks	AO	Guidance
6	(b)	cont	<p>If W and Z are chosen X has probability p and Y has prob $1 - p$ A: $4p + (1 - p) = 1 + 3p$ B: $0 + 4(1 - p) = 4 - 4p$ C: $p + 5(1 - p) = 5 - 4p$ D: $6p + 0(1 - p) = 6p$</p> <p>$5 - 4p = 6p (= \frac{1}{2})$</p>	<p>M1 ft</p> <p>M1 ft A0 A0</p>		<p>Or the negatives of these for the alternative method</p> <p>Definition of p may be implied for M marks Using their reduced table (or correct) and their (single) variable At least two correct expressions (need not be simplified) seen or implied from graph</p> <p>Or $4p - 5 = -6p$ for the alternative method</p>

Question		Answer	Marks	AO	Guidance															
7	(a)	 <table border="1" data-bbox="427 863 994 1058"> <thead> <tr> <th>x</th> <th>y</th> <th>$P = 4x + y$</th> </tr> </thead> <tbody> <tr> <td>6</td> <td>0</td> <td>24</td> </tr> <tr> <td>10.5</td> <td>1.5</td> <td>43.5</td> </tr> <tr> <td>11.25</td> <td>3.75</td> <td>48.75</td> </tr> <tr> <td>0</td> <td>15</td> <td>15</td> </tr> </tbody> </table> <p data-bbox="427 1107 555 1134">$P = 48.75$</p> <p data-bbox="427 1158 672 1185">$x = 11.25, y = 3.75$</p>	x	y	$P = 4x + y$	6	0	24	10.5	1.5	43.5	11.25	3.75	48.75	0	15	15	<p data-bbox="1043 347 1088 375">B1</p> <p data-bbox="1043 459 1088 486">B1</p> <p data-bbox="1043 571 1088 598">B1</p> <p data-bbox="1043 871 1088 898">M1</p> <p data-bbox="1043 1110 1088 1137">A1</p> <p data-bbox="1043 1161 1088 1189">A1</p> <p data-bbox="1043 1212 1088 1240">[6]</p>	<p data-bbox="1151 347 1196 375">1.1</p> <p data-bbox="1151 459 1196 486">1.1</p> <p data-bbox="1151 571 1196 598">1.1</p> <p data-bbox="1151 871 1196 898">1.1</p> <p data-bbox="1151 1110 1196 1137">1.1</p> <p data-bbox="1151 1161 1196 1189">1.1</p>	<p data-bbox="1238 347 1957 375">Boundary line $y = 3x - 30$, at least from (10, 0) to (12, 6)</p> <p data-bbox="1238 459 1935 486">Boundary line $y = 15 - x$, at least from (6, 9) to (12, 3)</p> <p data-bbox="1238 571 1935 598">Boundary line $y = \frac{1}{3}x - 2$, at least from (6, 0) to (12, 2)</p> <p data-bbox="1238 660 1518 687">Ignore any extra lines</p> <p data-bbox="1238 871 2013 975">Checking any vertex (other than the origin) or using a sliding profit line with gradient -4 (approx.) seen, or implied from answer</p> <p data-bbox="1238 1110 1285 1137">cao</p> <p data-bbox="1238 1161 1285 1189">cao</p>
x	y	$P = 4x + y$																		
6	0	24																		
10.5	1.5	43.5																		
11.25	3.75	48.75																		
0	15	15																		

Question			Answer	Marks	AO	Guidance
7	(b)	(i)	Additional constraint $y \geq kx$ $3.75 \div 11.25 = \frac{1}{3}$ Solution is unchanged for $k \leq \frac{1}{3}$	M1 A1ft [2]	3.1a 1.1	$\frac{1}{3}$ or (their y) \div (their x) \leq (their) $\frac{1}{3}$ Accept with a lower bound of 0, $0 < k \leq$ (their) $\frac{1}{3}$, provided their upper bound is positive, but not with a lower bound other than 0
7	(b)	(ii)	For $k > \frac{1}{3}$, optimal solution is at intersection of $y = kx$ and $x + y = 15 \Rightarrow x + kx = 15$ $P = 15\left(\frac{4+k}{1+k}\right)$ $x = 15\left(\frac{1}{1+k}\right)$ $y = 15\left(\frac{k}{1+k}\right)$	M1 A1 A1 A1 [4]	3.1a 1.1 1.1 1.1	Evidence of finding point where $y = kx$ meets $x + y = 15$ May be implied from any of x, y, P correct cao (or equivalent) cao (or equivalent) without wrong working cao (or equivalent) without wrong working

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